THE
AMERICAN
MATHEMATICAL MONTHLY.

VOL. I. FEBRUARY, 1894. NO. 2.

BIOGRAPHY.

PROFESSOR WILLIAM HOOVER, A. M., Ph. D.

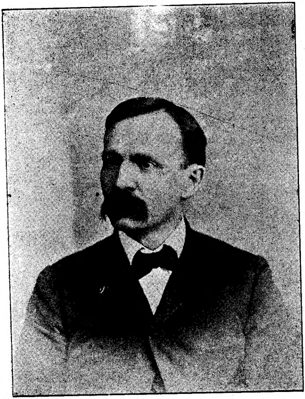
Professor Hoover was born in the village of Smithville, Wayne county Ohio, October 17, 1850, and is the oldest of a family of seven children. Both parents are living in the village where he was born, still enjoying good health.

Up to the age of fifteen he attended the public schools, and for two or three years after, a local academy. Owing to needy circumstances he was obliged to work for his living quite early, and almost permanently closed attendance at any kind of school at eighteen years of age, sometime before which, going into a store in the county seat, as clerk. Nothing could have been farther from his taste than this work, having been thoroughly in love with study and books long before. After spending two or three years in this way, he went to teaching, about the year 1869, and he has been regularly engaged in his favorite profession to the present day.

He attended Wittenberg College and Oberlin College one term each, a thing having very little bearing on his education. He studied no mathematics at either place, excepting a little descriptive astronomy at the latter.

After teaching three winters of country school, with indifferent success, he was chosen, in 1871, a teacher in the Bellefontaine, Ohio, High School, serving one year, when he was given a place in the public schools of South Bend, Ind. Remaining there two years, he was invited to return to Bellefontaine as superintendent of schools. He afterwards served in the same capacity in Wapakoneta, O., two years, and as principal of the second district school at Dayton, O. In 1883, he was elected professor of mathematics and astronomy in the Ohio University, Athens, Ohio, where he is still in service.

Through all his career of teaching, Professor Hoover has been an incessant student, devoting himself largely to original investigations in mathematics. Although his pretentions in other lines are very modest, he is eminently proficient in literature, language, and history. Before going into college work he



PROF. WILLIAM HOOVER, A. M., PH. D.

had collected a good library. He is indebted to no one for any attainments made in the more advanced of these lines, but by indefatigable energy and perseverance he has made himself the cultured, classic, and renowned scholar he is.

He has always been a thorough teacher, aiming to lead pupils to a mastery of subjects under consideration. His habits of mind and preparation for the work show him specially adapted to his present position, where he has met with great success. He studies methods of teaching mathematics, which in the higher parts is supposed to be dry and uninteresting. He sets the example of enthusiam as a teacher, and rarely fails to impress upon the minds of his students the immense and varied applications of mathematics. He is kind and patient in the class-room and is held in the highest esteem by his students. He is ever ready to aid the patient student inquiring after truth. It seems to be a characteristic of eminent mathematicians that they desire to help others to the same heights to which they themselves have climbed. This was true of Professor Seitz; it is true of Dr. Martin; and it is true of Professor Hoover.

In 1879, Wooster University conferred upon Professor Hoover the degree of Master of Arts, and, in 1886, the degree of Doctor of Philosophy *cum laude*, he submitting a thesis on Cometary Perturbations. In 1889, he was elected a member of the London Mathematical Society and is the only man in his state enjoying this honor. In 1890, he was elected a member of the New York Mathematical Society. He has been a member of the Association for the Advancement of Science for several years. Papers accepted by the association at the meeting at Cleveland, Ohio, and at Washington, D.C., have been presented on "The Preliminary Orbit of the Ninth Comet of 1886," and "On the Mean Logarithmic Distance of Pairs of Points in Two Intersecting Lines." He is in charge of the correspondence work in mathematics in the Chautauqua College of Liberal Arts and of the mathematical classes in the summer school at Lake Chautauqua the principal of which is the distinguished Dr. William R. Harper, president of the new Chicago University. The selection of Professor Hoover for this latter position is of the greatest credit, as his work is brought into comparison with some of the best done anywhere.

He is a critical reader and student of the best American and European writers, and besides, is a frequent contributor to various mathematical journals, the principal of which are *School Visitor*, *Mathematical Messenger*, *Mathematical Magazine*, *Mathematical Visitor*, *Analyst*, *Annals of Mathematics*, and *Educational Times* of London, England.

His style is concise and his aim is elegance in form of expression of mathematical thought. While greatly interested in the various branches of pure mathematics, he is specially interested in the applications to the advanced departments of Astronomy, Mechanics, and the Physical Sciences—such as Heat, Optics, Electricity, and Magnetism. The "electives" offered in the advanced work for students in his University are among the best mathematics pursued any where in this country.

He is an active member of the Presbyterian church and greatly interested in every branch of church work. He has been an elder for a number of

years and was chosen a delegate to the General Assembly, meeting at Portland, Oregon, in May, 1892, serving the church in this capacity with fidelity and intelligence. In this biography of Professor Hoover, there is a valuable lesson to be learned. It is this: energy and perseverence will bring a sure reward to earnest effort. We see how the clerk in a county-seat store, in embarassing circumstances and unknown to the world of thinkers, became the well known Professor of Mathematics and Astronomy in one of the leading Institutions of learning in the State of Ohio. "Not to know him argues yourself unknown."

[From *Finkel's Mathematical Solution Book.*]

APPLICATION OF THE NEW EDUCATION TO THE DIFFERENTIAL AND INTEGRAL CALCULUS.

By FLETCHER DURELL, Ph. D., Professor of Mathematics, Dickinson College, Carlisle, Pennsylvania.

[Continued from the January Number]

If the quantity which has been represented by curves may also be regarded as existing independently of any spacial arrangement, its magnitude and magnitude relations in both cases being the same, the formulas of differentiation obtained above apply to both; that is, they apply to functions as well as to curves. The student may at once be brought to realize the greater flexibility and freedom of treatment obtained by using them functionally. We thus arrive at the more general definition of differentiation, $\dfrac{dy}{dx}=\text{limit}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$.

The process of determining the slopes of curves by the above geometrical method, and the use of the related variables as auxiliary quantities determining the slopes by exact contact, and the practice in constructing tangents to the curves by the use of these slopes, build up firm and exact and vivid conceptions of the quantities dealt with. When the student comes to take up the more general idea of functional quantities, arranged irregularly or indefinitely in space, the geometrical conceptions already formed aid in giving firmness and reality to the quantities dealt with as differential coefficient, and a sense of the absolute precision of their values as determined by variables moving up into contact with them.

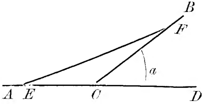
However at the outset of each division of the subject, as in dealing with partial differentiation, series, indeterminate expressions so-called, etc, it is best to establish properties in the geometrical form if only for the double light that is thrown on them. Space will not permit us to show in detail how this is done, and we will but illustrate these further applications of the method by giving a proof of Taylor's Formula with Remainder. In Fig. 5, let (I), or PQ, be the section of the surface, $u=f(x+y)$ made by the ux-plane. Since this surface slopes in the same way from the xy—plane, as it does from the uy—plane, this one trace may be taken as an adequate representation of the whole surface for the present purpose.

EASY COROLLARIES IN NON-EUCLIDEAN GEOMETRY.

By GEORGE BRUCE HALSTED, A. M. (Princeton), Ph.D. (Johns Hopkins), Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.

In that sort of Non-Euclidean geometry usually named after Lobatschewsky, it is readily demonstrated that the angle-sum of a rectilineal triangle is a variable directly connected with the size of the triangle, just as is the case in spherics. This proof in a very elementary form is given by John Bolyai. His section 41 is, "Equivalent triangles have their angle-sums equal." Then as an easy corollary, section 42, "Triangles are to each other as the supplements of their angle-sums."

From this we get, at once, the corollary, that in Lobatschewsky's geometry there may be a triangle whose angle-sum differs from a straight angle by less than any given finite angle however small.



For a single angle ACB can always be drawn less than a straight angle and in such manner as that it shall differ from a straight angle by as small an angle, a, as any given finite angle however small. Then drawing a straight line from any point E on the arm AC to any point F on the arm BC we shall have a triangle ECF, the supplement of whose angle-sum is less than a.

Hence the further corollary, that we can always draw a triangle less in size than any triangle whose angle-sum is less than a straight angle by the given finite angle a.

BIBLIOGRAPHY OF THE HISTORY OF GEOMETRY; ALSO A LIST OF MATHEMATICAL PERIODICALS.

By ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.

The following list was prepared in the belief that it would be of interest and value to those who are making a study of the history and developement of Geometry. The list does not pretend to be complete, yet it is thought that the principal English, French, and German works are included. The list of American Periodicals is thought to be complete. The date given in connection with each periodical is the date of first issue. In the case of periodicals that have been discontinued, the date of discontinuation, when possible, is given. The list has been made by a careful study of the references in the leading histories of Mathematics. The place of publication of a number of the periodicals has not been determined with certainty; these are left blank in the list. Corrections and additions will be gladly received by the writer.

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REMARKS ON DIVISION.

By J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

A large majority of the arithmetics in use in the United States teach that a "concrete number" can be divided by an abstract or pure number; that if the dividend is $10 and the divisor 2, the quotient is $5. Why this has been allowed to go unchallenged for so many generations is a psychological rather than a mathematical problem. Mathematicians have been neither scarce nor idle, but they seem to have been working upward and outward among the branches instead of digging down to the rootlets of the infinite tree of mathematical truth.

In the realm of number all the human mind can do is unite and take apart, involve and evolve, compose and analyze. Everthing is based upon addition. The inverse of additon is subtraction. Multiplication is a mere process of adding, hence *its* inverse is subtraction. If a given product is $20 and the multiplicand $5, we can find the multiplier by subtracting $5 from $20 until nothing remains. The number of times we subtract is 4, the multiplier. One number can be taken from another just as often as it is contained therein; hence, division is equivalent to subtraction, and is the inverse of multiplication.

If a given product is $20, and the multiplier 4, we cannot by mere subtraction find the multiplicand. That is to say, multiplication has but one in-

verse, which is subtraction or its equivalent, division. But by subtraction we can find only *how many times*; hence, by division the only thing that can be found is *how many times*. That is, how many times one number is contained in another of the same kind. It is thus seen that quotient is always *abstract*. Consequently, the dividend and divisor must be like numbers; for if a quotient is a, the dividend is a times the divisor, whatever it may be. Therefore, a "concrete number" can not be divided by an abstract one.

To find one of the equal parts of a concrete number is more than division: it is a problem that involves the use of division; it is an "application" of division, just as "profit and loss" is an application of percentage. Thus, to find $\frac{1}{4}$ of \$20, we proceed logically as follows:

(*a*) $\frac{1}{4}$ of \$20 is as many dollars as there are 4's in 20.

(*b*) There are *five* 4's in 20.

(*c*) $\therefore$ $\frac{1}{4}$ of \$20 is \$5.

The reasoning in such problems must be in the abstract, and the result interpreted or applied in the conclusion. But pure division involves none of this reasoning—it involves only a retracing of the steps in multiplication or addition.

It is plain that to find $\frac{1}{4}$ of a number is to divide that number by 4. To find $\frac{1}{4}$ of $\frac{8}{9}$ is to divide $\frac{8}{9}$ by 4. Hence, the alleged "compound fraction" is no fraction at all; it is not even an example in multiplication of fractions, as given by all arithmetics, but it is clearly an example in *division* of fractions.

Expressions like $\frac{4}{\frac{2}{3}}$, commonly called "complex fractions", are not fractions; they are indicated *divisions*. They have the *form* of a fraction, but so has an Indian tobacco sign the form of a man. Unexecuted division and ratio may be expressed in fractional form, but a fraction expresses neither division nor ratio. Thus, $8\div9$ may be written $\frac{8}{9}$; but this does not denote 8 of the nine equal parts of "a unit." When $\frac{8}{9}$ expresses a division to be performed, the 9 is a number—*nine;* when it is a fraction, the 9 is a name—*ninths*.

In the former case the expression is read 8 divided by nine; in the latter it is read 8 ninths.

Besides, if $\frac{4}{\frac{2}{3}}$ were a fraction, the denominator $\frac{2}{3}$ would indicate that some unit had been divided into $\frac{2}{3}$ equal parts! Since it is impossible for *man* to so divide a unit, this species of complex fractions must be regarded as a special gift from on high, "for with God all things are possible."

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him

5. Proposed by E. E. KINNEY, Anaconda, Montana.

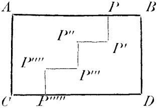
A board is 16 inches long and 9 inches wide. How may it be cut in two

parts that the parts joined together may form a square?

Solution by H. W. DRAUGHON, Clinton, Louisiana.

Let $ABCD$ be the board, and let the broken line $PP'P''P'''P''''P'''''$ be the line of division.

We make $BP=P'P''=P'''P''''=P'''''C=4$ inches, and all parallel to BP; also we make, $PP'=P''P'''=P''''P'''''=3$ inches, and all parallel to BD. From the construction it is obvious that if B be placed at P' and P'''' at C the resulting figure will be 1 foot square.



Solved in a similar manner by *G. B. M. Zerr*, *Robert J. Aley*, and *J. A. Calderhead*.

PROBLEMS.

12. Proposed by CHARLES E. MYERS, Canton, Ohio.

A man made his will to this effect: that if only the daughter returned home his wife should have $\frac{2}{3}$ and the daughter $\frac{1}{3}$ of the estate; and if only the son returned, his wife should have $\frac{1}{3}$ and the son $\frac{2}{3}$. But the son and daughter both returned. How should the estate be divided?

13. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics in the Davenport Business College, Davenport, Iowa.

A man borrowed \$5000 at a western bank giving his note for \$5000 due in 5 years without grace at 8% interest payable annually, and pays the banker a bonus of \$500 in cash for making the loan; what rate per cent. does he pay?

[Solutions to these problems should be received by April 1st.]

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

9. Proposed by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

$$\left.\begin{array}{l} x+y^2+z^2=21 \\ x^2+y^3+z=45 \\ x^3+y+z^2=71 \end{array}\right\}. \text{ Find } x,\ y, \text{ and } z.$$

10. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania,

$$x^2+y^2+w^2+z^2=65\ldots.(1),$$
$$(x+z)^2+(y+w)^2=113\ldots.(2),$$
$$(y+z)^2+(x+y)^2=117\ldots.(3),$$
$$(x+y)^2+(z+w)^2=125\ldots.(4).$$

How many values has each of the four unknown quantities?

11. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Two men, A and B, had a money-box, containing \$210, from which each drew

a certain sum daily; this sum being fixed for each, but different for the two. After six weeks, the box was empty. Find the sum which each man drew daily from the box: knowing that *A alone* would have emptied it five weeks earlier than *B alone*.

12. Proposed by F. M. SHIELDS, Coopwood, Mississippi.

Three lads, A, B, and C, each climbed to the top of an upright pole: A's pole was 20 feet high, B's 60 feet, and C's pole was 100 feet high. They all started at the same time, and each climbed up a part of the way, at the same rate of speed per minute, and after each rested 5 minutes, they ascended to the tops of their respective poles, at the same rate of speed per minute, when they found that each had consumed the same length of time, 25 minutes each, (including the 5 minutes each rested on the way). How far up did each climb before resting? At what rates of speed per minute did they ascend?

13. Proposed by H C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Six city boys, Jim, Josh, Jerry, Jack, Jake and Jeorje went into the country to steal apples from a tree. While three kept watch, the other three climbed up and got what they wanted. Then they came down while the other three rascals went up and stole. The one that got most was one of the last to go up.

Each trio of theives took the same number and had each boy taken as many as he did take in *each of that number of pockets*, each trio would also have taken the same number and the tree would have lost 538 apples. As it was, Josh got more than Jack, but Jeorje got as many as Josh and Jack together, while Jake got twice as many as Jerry and two more than Jim. What were the names of the three that first kept watch? [Figures altered from problem in *Henkle's Notes and Queries.*]

[Solutions to these problems should be received by April 1st.]

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

2. Show that $\frac{1}{2}\pi=\left\{\frac{2.\ 4.\ 6.\ 8.\ 10}{1.\ 3.\ 5.\ 7.\ 9}\cdots\cdots\right\}^2$, *Wallis's* expression for π.

[Selected from *Bowser's Trigonometry.*]

Solution by Professor G. B. M. ZERR. A. M., Principal of School, Staunton, Virginia.

All trigonometries solve the following:

$$\sin\ \theta=\theta\left\{1-\frac{\theta^2}{\pi^2}\right\}\left\{1-\frac{\theta^2}{2^2\pi^2}\right\}\left\{1-\frac{\theta^2}{3^2\pi^2}\right\}\left\{1-\frac{\theta^2}{4^2\pi^2}\right\}\cdots\cdots$$

Now let $\theta=\frac{\pi}{2}$. Then $1=\frac{\pi}{2}(1-\frac{1}{4})(1-\frac{1}{16})(1-\frac{1}{36})(1-\frac{1}{64})(1-\frac{1}{100})(1-\frac{1}{144})\cdots$

$$=\frac{\pi}{2}\cdot\frac{3}{4}\cdot\frac{15}{16}\cdot\frac{35}{36}\cdot\frac{63}{64}\cdot\frac{99}{100}\cdot\frac{143}{144}\cdots$$

$$=\frac{\pi}{2}\cdot\frac{3}{2.2}\cdot\frac{3.5}{4.4}\cdot\frac{5.7}{6.6}\cdot\frac{7.9}{8.8}\cdot\frac{9.11}{10.10}\cdot\frac{11.13}{12.12}\cdots$$

$$=\frac{\pi}{2}\cdot\frac{3^2.5^2.7^2.9^2.11^2}{2^2.4^2.6^2.8^2.10^2}\cdots$$

$$\therefore\ \frac{\pi}{z}=\left\{\frac{2.4.6.8.10}{1.3.5.7.9.11.}\cdots\right\}^2$$

3. If A be the area of the circle inscribed in a triangle, A_1, A_2, A_3 the areas of the escribed circle, show that $\dfrac{1}{\sqrt{A}}=\dfrac{1}{\sqrt{A_1}}+\dfrac{1}{\sqrt{A_2}}+\dfrac{1}{\sqrt{A_3}}$.

[Selected from *Todhunter's Plane Trigonometry.*]

Solution by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana State University, Bloomington, Indiana, and J. A. CALDERHEAD, Superintendent of School, Limaville, Ohio.

$r=\dfrac{S}{s}$, where r=radius of inscribed circle, S=area of triangle, and $s=\frac{1}{2}(a+b+c)$=half the sum of the sides of triangle. (*Todhunter's Plane Trigonometry*, Art. 248.)

Also $r_1=\dfrac{S}{s-a}$, $r_2=\dfrac{S}{s-b}$, $r_3=\dfrac{S}{s-c}$ where r_1, r_2, r_3 respectively, represent the radii of the escribed circles. (Art. 250).

$\therefore A=\dfrac{\pi S^2}{s^2}, A_1=\dfrac{\pi S^2}{(s-a)^2}, A_2=\dfrac{\pi S^2}{(s-b)^2}, A_3=\dfrac{\pi S^2}{(s-c)^2}$.

$$\therefore \frac{1}{\sqrt{A}}=\frac{s}{S\sqrt{\pi}},\ \frac{1}{\sqrt{A_1}}=\frac{s-a}{S\sqrt{\pi}},\ \frac{1}{\sqrt{A_2}}=\frac{s-b}{S\sqrt{\pi}},\ \frac{1}{\sqrt{A_3}}=\frac{s-c}{S\sqrt{\pi}}.$$

$$\therefore \frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}=\frac{s-a}{S\sqrt{\pi}}+\frac{s-b}{S\sqrt{\pi}}+\frac{s-c}{S\sqrt{\pi}}=\frac{3s-(a+b+c)}{S\sqrt{\pi}}=\frac{s}{S\sqrt{\pi}}.$$

But $\dfrac{1}{\sqrt{A}}=\dfrac{s}{S\sqrt{\pi}}$. $\therefore \dfrac{1}{\sqrt{A}}=\dfrac{1}{\sqrt{A_1}}+\dfrac{1}{\sqrt{A_2}}+\dfrac{1}{\sqrt{A_3}}$. Q. E. D.

Also solved by *G. B. M. Zerr*, *P. S. Berg*, *P. H. Philbrick*, *J. R. Baldwin*, and *H. C. Whitaker*.

PROBLEMS.

17. **Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.**

Draw a circle bisecting the circumferences of three given circles.

18. **Proposed by Professor HENRY HEATON, Atlantic, Iowa.**

Through two given points to draw two circles tangent to a given circle.

19 **Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.**

If any point be taken in the circumference of a circle, and lines be drawn from it to the three angles of an inscribed equilateral triangle, prove that the middle line so drawn is equal to the sum of the other two.

20. **Proposed by GEORGE BRUCE HALSTED, A. M., Ph. D., Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.**

Demonstrate by pure spherical geometry that spherical tangents from any point in the produced spherical chord common to two intersecting circles on a sphere are equal.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

7. Proposed by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

To determine the function $F(x)$ so that $F(x+y)\times F(x-y)=[F(x)]^2-[F(y)]^2$.

8. A woodman fells a tree 2 feet in diameter, cutting half way through from each side. The lower face of each cut is horizontal, and the upper face makes an angle of 45° with the horizontal. How much wood does he cut out?

[Selected from *Byerly's Integral Calculus*.]

9. Proposed by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

The solids bounded by the surfaces whose equations are $\left|\frac{x}{a}\right|^{\frac{2}{3}}+\left|\frac{y}{b}\right|^{\frac{2}{3}}+\left|\frac{z}{c}\right|^{\frac{2}{3}}=1$, and $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ where $a>b>c$ have their centers coincident. Find (1 and 2) the volume of each without the other, and (3) the volume common to both by direct integration, using the formula $V=\int\int\int dxdydz$.

10 Proposed by ERIC DOOLITTLE, Instructor in Mathematics. State University of Iowa.

Prove or disprove the following theorem: If O be any circle, and AB any straight line either within or without the circumference, and if a perpendicular be dropped from O upon AB and prolonged backward to meet the circumference in P, *then* will the angle whose vertex lies at P and whose sides pass through A and B, cut a portion CPD from the circle which shall be greater than that cut by any other angle whose vertex lies on the circumference and whose sides pass through A and B.

[If any one can give a solution without the use of the Calculus, it will also be acceptable —ED.]

11. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

A ribbon, 1 inch wide is wrapped spirally around a right prism, altitude 10 ft., bases of n sides, radius of circumscribed circle, 1 ft., so as to cover the entire convex surface. (1) What is the length of the ribbon? (2) If the ribbon is unwound and kept tense, by a power acting on the lower end, and moving in the plane of the lower base, what will be the length of the curve described by the power?

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

PROBLEMS.

6. Proposed by THOMAS W. WRIGHT, M. A., Ph. D., Professor of Applied Mathematics and Physics, Union College, Schenectady, New York.

What is the effect of a charge between light and heavy cavalry, the light cavalry having the greater energy and the heavy the greater momentum?

7. **Proposed by DE VOLSON WOOD, M. A., M. Sc., C. E., Professor of Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

A hollow sphere filled with frictionless water rolls down a rough plane whose length is l and inclination θ; when half way down the water suddenly freezes and adheres to the sphere. Required the time of the descent.

8. **Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.**

A heavy bar AB of length a falls about its lower end B from a vertical to a horizontal position, when the end A is suddenly fixed and B is set free, so that the bar falls into a vertical position AB as at first; then A is set free, and B is fixed, so that the bar again falls about B into a horizontal position, when the end A is suddenly fixed, and B is set free, and so on; find the angular velocity ω of the bar about the upper end, when it takes a vertical position for the nth time.

[Selected from *Price's Infinitesimal Calculus.*]

SOLUTION TO THE CELEBRATED INDETERMINATE EQUATION.

$$x^2-Ny^2=\pm1.$$

To obtain the values of x and y, in integers, without taking the square root of N, by continued fractions. On account of requiring a certain class of non-quadrate numbers for N, to satisfy the -1, and causing more or less confusion, this part is set aside for future time. The $+\,1$ is found in all values of N, and our equation becomes $x^2=Ny^2+1$.

Then $Ny^2+1=\square=\left(\dfrac{mNy}{n}-1\right)^2$ when reduced easily gives

$$y=\frac{2mn}{m^2N-n^2}, \quad \text{and } x=\frac{mNy}{n}-1, \ldots\ldots (\text{A})$$

Let $\dfrac{m}{n}=\dfrac{1}{2}$, then $y=\dfrac{4}{N-4}$. Let

$N=$	2,	3,	$\square$,	5,	6,	8,	$\square$
then $y=$	2	4		4	2	1	
$x=$	3	7		9	5	3	

$\dfrac{m}{n}=\dfrac{1}{3}$, and $y=\dfrac{6}{N-9}$,

$N=$	3,	7,	8,	10,	11,	12,	(13),
$y=$	1	3	6	6	3	2	
$x=$	2	8	17	19	10	7	

$\dfrac{m}{n}=\dfrac{1}{4}$, and $y=\dfrac{8}{N-16}$,

$N=$	14,	15,	$\square$,	17,	18,	(19),	20,
$y=$	4	8		8	4		2,
$x=$	15	31		33	17		9

$\dfrac{m}{n}=\dfrac{1}{5}$, $y=\dfrac{10}{N-25}$,

$N=$	15,	(21),	(22),	23,	24,	$\square$,	26,	27,	(28),	(29),	30,
$y=$	1			5	10		10	5			2
$x=$	4			24	49		51	26			11

$\dfrac{m}{n}=\dfrac{1}{6}$, $y=\dfrac{12}{N-36}$,

$N=$	24,	(31),	32,	33,	34,	35,	$\square$,	37,	38,	39,	40,	(41),	42,
$y=$	1		3	4	6	12		12	6	4	3		2
$x=$	5		17	23	35	71		73	37	25	19		13

$\frac{m}{n}=\frac{1}{7}$, $y=\frac{14}{N-49}$, N=35,(43),(44),(45),(46),47,48, □ ,50,51,(52),(53),(54),(55),56,

y=1 ... 7 14 ... 14 7 ... 2

x=6 ... 48 97 ... 99 50 ... 15

$\frac{m}{n}=\frac{1}{8}$, $y=\frac{16}{N-64}$, N=48,57,58,59,60,61,62,63, □ ,65,66,67,68,69,70,71,72,

y=1 ... 4 ... 8 16 ... 16 8 ... 4 ... 2

x=7 ... 31 ... 63 27 129 ... 65 ... 17

$\frac{m}{n}=\frac{1}{9}$, $y=\frac{18}{N-81}$, N=63,73,74,75,76,77,78,79,80, □ ,82,83,84,85,86,87,88,89,90,

y=1 ... 3 ... 6 9 18 ... 18 9 6 ... 3 ... 2

x=8 ... 19

$\frac{m}{n}=\frac{1}{10}$, $y=\frac{20}{N-100}$, N=80,91,92,93,94,95,96,97,98,99, □ ,101,102,103,104,105,106,107,108,110,

y=1 ... 4 5 ... 10 20 20 ... 10 ... 5 4 ... 2

x=

It is seen that the position of the numbers N, between the two consecutive square numbers, determines the ease of obtaining the values of x and y. As, let a^2 and a_2^2, represent the two squares, then in $a^2+a, y=2$, in numbers N, of the form of $a^2+2a, y=1$. The more complex numbers are always found midway between a^2+a and a^2+2a.

There are two values of x and y, N being of the form of a^2-1. It is, however, the second higher value of x and y, as x_2 and y_2.

Other series can easily be found giving, the values also in a regular series.

A. H. BELL, Hillsboro, Illinois.

(TO BE CONTINUED.)

AVERAGE AND PROBABILITY

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

Solution to a Problem by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

Three persons $A.$, $B.$, $C.$, throw with three dice. They each stake $10, and the one who first throws exactly 10 with the three dice, takes the whole stake. Find the expectation of each.

There is probably no subject that is more interesting than the subject of Average and Probability, and that part of this subject that deals

with games of chance is especially fascinating. In the above problem in order to throw ten, one die must turn up 6, one 3, and one 1; or one must turn up 6, one 2,and one 2;or one must turn up 5,one 3,and one 2;or one must turn up 5,one 4,and one 1;or one must turn up 4, one 4,and one 2; or one must turn up 4,one 3, and one 3. The first can happen six ways as follow: 6, 3, 1; 6, 1, 3; 3, 1, 4; 3, 1, 6; 1, 3, 6; 1,6, 3;.. in all 6 ways,

the second 6, 2, 2; 2, 6, 2; 2, 2, 6;.......................... in all 3 ways,

the third 5, 3, 2; 5, 2, 3; 2, 5, 3; 2, 3, 5; 3, 2, 5; 3, 5, 2;.... in all 6 ways,

the fourth 5, 4, 1; 5, 1, 4; 4, 1, 5; 4, 5, 1; 1, 5, 4; 1, 4. 5;.....in all 6 ways,

the fifth 4, 4, 2; 4, 2, 4; 2, 4, 4;.............................in all 3 ways,

the sixth 4, 3, 3; 3, 4, 3; 3, 3, 4;............................in all 3 ways,

GIVING 27 WAYS.

The chance that one die will turn up 6 is $\frac{1}{6}$.

The chance that another die will turn up 3 is $\frac{1}{6}$.

The chance that third die will turn up 1 is $\frac{1}{6}$.

Hence, the chance that 6, 3, 1 in order given will turn up on one throw is $\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=(\frac{1}{6})^3$ and since ten can turn up in 27 ways, the chance of throwing ten on the first throw is $27\times(\frac{1}{6})^3=(\frac{3}{6})^3=(\frac{1}{2})^3=\frac{1}{8}$.

$\therefore$ The chance that A will throw ten is $\frac{1}{8}$ and the chance that he will not throw ten is $\frac{7}{8}$. The chance then that B will throw ten is the product of the chances of A's failing and of throwing ten on one throw $=\frac{1}{8}\times\frac{7}{8}$; the chance of A and B both failing is $1-(\frac{1}{8}\times\frac{7}{8}+\frac{1}{8})=\frac{49}{64}=(\frac{7}{8})^2$: unity minus the chance that A throws ten plus the chance, if A fails, that B throws ten .

Hence, the chance that C throws ten is $\frac{1}{8}\times(\frac{7}{8})^2$ and the chance that he fails is $1-\left\{\frac{1}{8}+\frac{1}{8}(\frac{7}{8})+\frac{1}{8}(\frac{7}{8})^2\right\}=(\frac{7}{8})^3$.

	A's chance	B's chance	C's chance
A's first throw......	$\frac{1}{8}$	$\frac{1}{8}(\frac{7}{8})$	$\frac{1}{8}(\frac{7}{8})^2$
A's second throw....	$\frac{1}{8}(\frac{7}{8})^3$	$\frac{1}{8}(\frac{7}{8})^4$	$\frac{1}{8}(\frac{7}{8})^5$
A's third throw.....	$\frac{1}{8}(\frac{7}{8})^6$	$\frac{1}{8}(\frac{7}{8})^7$	$\frac{1}{8}(\frac{7}{8})^8$
&c. &c. &c.	&c.	&c.	&c.

A's chance is the sum of the geometrical progression $\frac{1}{8}\left\{1+\left(\frac{7}{8}\right)^3\right.$

$\left.+\left(\frac{7}{8}\right)^6+\left(\frac{7}{8}\right)^9+\ldots\ldots+\text{to infinity}\right\}=\frac{1}{8}\times\frac{(8)^3}{169}=\left(\frac{8}{13}\right)^2$

B's chance is the sum of the geometrical progression $\frac{1}{8}\left\{\frac{7}{8}+\left(\frac{7}{8}\right)^4\right.$

$\left.+\left(\frac{7}{8}\right)^7+\left(\frac{7}{8}\right)^{10}+\ldots\ldots+\text{to infinity}\right\}=\frac{7\times8}{(13)^2}=\frac{56}{(13)^2}$.

C's chance is the sum of the geometrical progression $\frac{1}{8}\left\{\left(\frac{7}{8}\right)^2\right.$

$\left.+\left(\frac{7}{8}\right)^5+\left(\frac{7}{8}\right)^8+\left(\frac{7}{8}\right)^{11}+\ldots\ldots+\text{to infinity}\right\}=\left(\frac{7}{13}\right)^2$.

The expectation in a game of chance is the product of the chance of winning into the stake offered minus the stake put up by the winning person.

A's expectation is $\left[\frac{8}{13}\right]^2$ of \$30—\$10=\$11$\frac{61}{169}$ —\$10=\$1$\frac{61}{169}$.

B's expectation is $\left[\frac{56}{(13)^2}\right]$ of \$30—\$10=\$9$\frac{159}{169}$—\$10=—\$$\frac{10}{169}$.

C's expectation is $\left[\frac{7}{13}\right]^2$ of \$30—\$10=\$8$\frac{118}{169}$—\$10=—\$1$\frac{51}{169}$.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

5. **Proposed by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.**

A cubic mile of saturated air at 18°C. is cooled to a temperature of 10°C. How many tons of rain will fall?

6. **Proposed by H. C WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

Two men wish to buy a grindstone 42 inches in diameter and one foot thick at the center. To what thickness at the outer edge should the stone uniformily taper from the center that each man may grind off 18 inches of the diameter and both have equal shares, the central six inches of the diameter being waste?

7. **Proposed by Rev. A. L. GRIDLEY, Pastor of Congregational Church, Kidder, Missouri.**

Making no allowance for the curvature of the earth and supposing the sun to rise in the east and set in the west, what would be the course of a man who should walk constantly toward the sun from morning until night? How far and in what direction from the starting point would he be, walking three miles per hour, at the end of three days?

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

Note to Professor Dickson's Article on Triangles, by Josiah H. Drummond, Portland, Maine.

The late Judge, Josiah Scott, of the Supreme Court of Ohio, published in 1871 a pamphlet (in which he modestly said, in substance, that while the Mathematician might find nothing new in it, yet the Student might not find it wholly without interest) demonstrated that $2mn$, m^2-n^2, and m^2+n^2 give the "lowest intergers representing sides of a right angled triangle" when m and n are any numbers, one odd and one even, and *prime to each other* and $m>n$.

This rule has seemed to me the most practical of any that I have seen.

We kindly call the attention of our contributors to the following: In making diagrams, draw them on white unruled paper and attach the same to the solution. Also, please, draw the diagram on as small a scale as possible, the large cuts being more expensive.

George B. Halsted, A. M.,(Princeton);Ph. D., (Johns Hopkins University), Professor of Mathematics in the University of Texas, has kindly consented to contribute a series of articles on Non-Euclidean Geometry, in future numbers of the MONTHLY. These articles alone will be worth many times the cost of the Journal, as Professor Halsted is recognized as one of the ablest expounders of the Non-Euclidean doctrine in the world.

Robert J. Aley, A. M., Professor of Mathematics in the University of Indiania, showed his appreciation of the MONTHLY by sending us the first club of fourteen subscribers.

Superintendent J. M. Greenwood, of Kansas City, Missouri, seems to have had great faith in THE AMERICAN MATHEMATICAL MONTHLY from the first; for no sooner did he receive our circular sent out in November than he immediately remitted, being the first to give the MONTHLY financial encouragement.

THE AMERICAN MATHEMATICAL MONTHLY is a fixed fact. We trust our friends will exert themselves to make it a financial success by securing new subscribers for us. Let each of our subscribers secure us a club of 10, yes even 1 new subscriber, and then we will soon be free from any anxiety of material loss in the publication of the MONTHLY.

BOOKS AND PERIODICALS.

Algebraische Gleichungen nebst den Resultaten und den Methoden zu ihrer Aufloeung. (Algebraic Equations with answers and methods of solution). by Dr. Ernst Bardy, Leipzig, 1893. Octavo, 378 pages.

This is the fourth edition of a well known German solution book for the use of teachers. It contains exactly one thousand exercises,all of which lead to a final quadratic equation. There are 492 examples involving one unknown quantity, 394 including two, and 114 embracing three or four unknowns. Answers are stated in all cases, and solutions of the more difficult exercises. Many special devices showing much engenuity on the part of the author are given, but it is a question if some examples are not of a nature to involve a great waste of time on the part of scholars. The following is one of the more difficult exercises:

$$\frac{y(1+x^2)}{x(1+y^2)}=a \qquad\qquad \frac{y^4(1+x^8)}{x^4(1+y^8)}=b$$

and the solution gives the results,

$$x=\tfrac{1}{2}(a\sqrt{u}+\sqrt{a^2u-4}) \quad y=\tfrac{1}{2}(\sqrt{u}+\sqrt{u-4})$$

in which u is a function of a and b which we leave to be deduced by young algebraists who are interested in problems of this kind. *Mansfield Merriman.*

Elementary Synthetic Geometry of the Point, Line, and Circle in the Plane. By N. F. Dupuis, M. A., F. R. S. C., Professor of Mathematics in the University of Queen's College, Kingston, Canada. 16 mo, cloth, 296 pp. Price, $1.10. New York: Macmillan & Co.

This work differs from the majority of treatises on Geometry in that it systematically paves the way for the study of Cartesian Geometry. The point, the line, and the curve lying in a common plane are taken as the geometric elements of Plane Geometry, and any one of these or any combination of them is defined as a geometric plane figure. The book is divided into five parts. Part I. treats of lines, angles, triangles, parallels, and the circle; Part II. treats of comparison of areas, measurements of areas, geometric interpretation of Algebric forms, areal relations, and squares and rectangles; Part III. treats of proportion amongst line-segments, functions of angles and their applications in geometry; Part IV. treats of geometric extensions, centre of mean position, colinearity and concurrence, inversion and inverse figures, pole and polar, radical axis, centers and axes of perspective or similitude; Part V. treats of anharmonic division, harmonic ratio, anharmonic ratio, polar reciprocals and reciprocation, homography and involution. Parts I. and II., each conclude with Constructive Geometry in which the preceding principles are applied to numerous interesting and well selected problems. In this geometry, are demonstrated a number of propositions not found in many other treatises on geometry. We do not hesitate to pronounce it a very fine work and teachers who are contemplating a change of texts in elementary geometry will do well to examine this little book before making a selection. B. F. F.

Complete Graded Arithmetic, Part Second, for Sixth, Seventh, and Eighth Grades. By George E. Atwood. 8vo, half leather back, 382 pp. Price,—Boston: D. C. Heath & Co

This arithmetic, which has just been issued, is one of the best that has yet appeared, for common and graded school work. It is abundantly supplied with well chosen problems, each one requiring the exercise of different powers of the mind. So many authors, in stating problems, have a frame-work of words in which at certain places numbers are inserted and thus their different problems are enunciated. This mistake Mr. Atwood has happily avoided.

The definitions and rules are resigned to the close of the book. This, to my mind, is an objection, but, by many teachers, this feature of the book will be highly appreciated. B. F. F.

A Manual of Land Surveying, comprising an Elementary Course of Practice with Instruments and a Treatise upon the Survey of Public and Private Lands, prepared for the use of Schools and Surveyors, by F. Hodgman, M. S., C. E., Practical Surveyor and Engineer, and C. F. R. Bellows, M. A., C. E., Professor of Mathematics in the Michigan State Normal School. Fifth Edition, Revised by F. Hodgman, 374 pp. + 112 pp. of tables. Price, $2.50. F. Hodgman, Climax, Michigan, 1891.

Besides treating satisfactorily the subjects usually considered in elementary works on Surveying, the book before us recognizes the fact that in a much larger part of the U.S., land surveys are made upon the U.S. land system, and aims to supply that which text-books are deficient in by discussing very fully U. S. Land Surveys and the many perplexing questions of practice arising under them.

The rules which govern resurveys are given a prominent place and the decisions of the highest courts are fully quoted on all important points relative to surveys. This is a special feature of this work.

The general problem of correcting a random line of several courses between two known points, in order to reproduce the original location of the line is given in

this manual. The problem arises in retracing meander lines, highway lines, boundaries of irregular tracts of land, and in all similar cases, and is of great practical importance, and yet few if any works on surveying make any reference to it. Land surveyors will appreciate the practical character of this work. To better fit it for a book of reference on the field it is brought within convenient pocket size by closely printing it on thin tough paper, with narrow margins. It is bound in flexible morocco with flaps. J. M. C.

Elementary Synthetic Geometry. By George Bruce Halsted, A. B., A. M., and ex-Fellow of Princeton College; Ph.D., and ex-Fellow of Johns Hopkins University; Professor of Mathematics in the University of Texas. Second edition, 1893. 8vo, cloth. Price, $1.50. New York: John Wiley & Sons.

For originality of treatment and logical arrangment, this work is superior to any other geometry in the language; it is geometry in a nut-shell. Dr. Halsted's introduction of new terms in the language of geometry is highly commendable, as such terms as *sect*, *straight*, *steregon*, etc., have become an absolute necessity to the progressive and logical teacher of geometry. An examination of the work is necessary in order to appreciate all its merits. B. F. F.

The Maine Farmers' Almanac, Charles E. Nash, Augusta, Maine, contains, besides the *calendar* and other interesting matter, a Puzzle Department and a Mathematical Department. The number for 1894 contains solutions to the four questions proposed last year and six new ones are proposed for the next number.

The Journal of Education, one of the leading weekly Journals for teachers, has an interesting Mathematical column in one issue of each month conducted by F. P. Matz, Ph. D., of Reading, Pennsylvania. In the issue of January 28, four interesting problems are solved.

The School Visitor, John S. Royer, Editor, Versailles, Ohio, enters upon its 15th volume. The January number contains solutions to 7 problems and a list of 17 very interesting problems are given for solution in future issues.

We are duly thankful for Professor Royer's kind reference to the MONTHLY.

The Educational Times, London, England, for January, has the usual amount of valuable sholastic matter and maintains the strength of its Mathematical Department. Fourteen problems are solved, and 36 new ones proposed. We are pleased to exchange with this valuable periodical.

Problem 12186 is from our valued contributor, Professor Zerr.

The Kansas University Quarterly for January, 1894, is at hand. The following is the Table of contents:

Report on Field Work in Geology by Erasmus Haworth, M. E. Kirk, and W. H. H. Piatt. A Geological Reconnoissance in Southwest Kansas and No Man's Land by E. C. Case; Traces of a Glacier at Kansas City, Missouri, by E. C. Case: New Genera and Species of Dolichopodidæ, by J. M. Aldrich; and Descriptions of North American Trypetidæ with Notes. Part I. by W. A. Snow.

No Mathematical papers are given in this number.

Annals of Mathematics for January, 1894, contains several very interesting articles, as follows:

A Construction for the Imaginary Points and Branches of Plane Curves, by F. H. Lond; The Screw as a Unit in a Grassmanian System of the Sixth Order, by E. W. Hyde: On the Theory of Functions of a Complex Variable, by W. H. Echols; Ziwet's Mechanics, by W. M. T.; Rules for the Algebraic Signs of Hyperbolic Formalae, by G.

Nacloskie; On the Descending Series for Bessel's Functions of both kinds, by James McMahon; An Elementary Deduction of Taylor's Formulae, by W. H. Echols: On Gauss's Method of Elimination, by Asaph Hall. One problem is proposed.

The Bulletin of the New York Mathematical Society for January, has for its leading Article, Modern Mathematical Thought by Professor Simon Newcomb. The other article is Recent Researches in Electricity and Magnetism, by Lieutenant Geo. O. Squier, U. S. A. Notes, and New Publications, fill the last pages. J. M. C.

Several Notices of books and pamphlets intended for this number will appear in the March issue.

Miscellaneous Notes and Queries, a Monthly Magazine of History, Folk-Lore, Mathematics, Mysticism, Art, Science etc., published by S. C. and L. M. Gould, Manchester, New Hampshire, edited by S. C. Gould. Price, $1.00 per year.

We have received the February and March numbers of this Monthly and find it truly full of things curious, quaint and olden, entering into every branch of science. literature, and art. It is a store-house of information to the student desiring many things from many sources. H. C. F.

Education, a Monthly Magazine devoted to the Science, Art, Philosopy, and Literature of Education, published by Kasson & Palmer, 50 Broomfield Street, Boston. Price, $3.00.

The January number of *Education* is before us and presents an entertaining and instructive table of contents,consisting of Secondary Education of Girls in France. A very strong paper on The Unconscious Element in Dicipline, by Henry S. Baker of St Paul, Minnesota. Every one who has made the subject a study will agree that Mr. Baker has touched the key note of discipline in school government. Another especially interesting article is on Western Reserve University, by Emerson O. Stevens. Cleveland, Ohio. Other features are Drawing in General Education, Shortened Writing, State University Library Work, Editorials, Echoes from the Exposition, Department of Professional Study, Foreign Notes, Among the Books, and Periodicals. This Magazine is international in its scope and therefore commends itself to the liberal educator. H. C. F.

ERRATA.

Vol. I., No. 1., p. 11, for John L. Lyle, Westminister College, read John N. Lyle, Westminster College.

p. 13, for Long Branch Depot, read North Branch Depot.

p. 19, solution to problem 2, step 2, for $\frac{5}{6}=\frac{5}{6}\times\frac{11}{11}=\frac{11}{\frac{55}{6}}=\frac{11}{9\frac{1}{6}}$ read $\frac{5}{6}=\frac{11}{11\div\frac{5}{6}}=\frac{11}{\frac{66}{5}}=\frac{11}{13\frac{1}{5}}$.

p. 21, first line in solution for opperations, read operations.

p. 23, problem 7, last line, for $(y-2mn)^2$, read $(y-2\,am)^2$.

p. 25, problem 3, for *total length swept over*, read total area swept over.